Tight Bounds for Single-Pass Streaming Complexity of the Set Cover Problem

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Joint work with Sanjeev Khanna (Penn) and Yang Li (Penn)

The Set Cover Problem

- Input: A collection of m sets $S_1, \ldots, S_m$ from a universe $[n]$.
- Goal: Choose a smallest subset C of the sets from $S_1, \ldots, S_m$ such that C covers $[n]$, i.e., $\bigcup_{i \in C} S_i = [n]$.

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Approximation vs Estimation:

- α-approximation: output a set cover of size at most $\alpha \cdot \mathsf{OPT}$ plus a certificate of coverage for each element $e \in [n]$.
- α-estimation: output an estimate for the size of minimum set cover in range $[\mathsf{OPT}, \alpha \cdot \mathsf{OPT}]$.

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- This talk: space complexity of approximating the set cover problem in the streaming model.

The Streaming Set Cover Problem

Model:

- The input sets $S_1, \ldots, S_m$ are presented one by one in a stream.
- The streaming algorithm has a small space to maintain a summary of the input sets.
- At the end, the algorithm outputs an exact/approximate set cover using this summary.

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Introduced originally by [SG09] and further studied in several recent works [ER14, DIMV14, IMV15, CW16, HPIMV16].

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Remark. We are not concerned with poly-time computability in this model.

State of the Art for Single-Pass Algorithms

Result	Space	Performance Ratio
Exact	$O(mn)$	1
[IMV15]	$\Omega(mn)$	$3/2 - \epsilon$
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Many known results for multi-pass algorithms as well: [SG09, IMV15, CW16] ...

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Single-pass Algorithms:

- $o(m)$ space regime is settled by the results of [ER14].
- However, sublinear space regime, that is, what can be done in $o(mn)$ space is wide open.
  - For example, is $O(1)$ approximation possible in $o(mn)$ space?
  - In general, what is the space-approximation tradeoff in this regime?

Our First Result

A tight space-approximation tradeoff for single-pass streaming algorithms:

Theorem

For any $\alpha = o(\sqrt{n})$, $\widetilde{\Theta}(mn/\alpha)$ *space is both sufficient and necessary for* α*-approximating the set cover problem.*

α-Approximation in $\widetilde{O}(mn/\alpha)$ space

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Our lower bound shows that this simple algorithm is essentially the best possible in terms of space requirement!

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However, our $\Omega(mn/\alpha)$ lower bound strongly relies on the fact that we are solving the approximation problem and not simply estimating the value of the optimal set cover.

Question: Can it be that estimation is strictly easier than approximation?

Our Second Result

Estimation is indeed distinctly easier!

Theorem

For any $\alpha = o(\sqrt{n})$, there exists a randomized α-estimation $\widetilde{O}(mn/\alpha^2)$ space algorithm for the streaming set cover problem.

Works in general for any covering integer program, and in particular for weighted set-cover or set multi-cover problem.

Our Third Result

The factor α gap between space requirements of approximation versus estimation algorithms for streaming set cover is tight.

Theorem

For any $\alpha = o(\sqrt{n})$, any randomized algorithm that α-estimates the set cover problem requires $\widetilde{\Omega}(mn/\alpha^2)$ space.

This lower bound holds even for random arrival streams.

$\Omega(mn/\alpha)$ Space is Necessary to Compute an α-Approximate Set Cover

Communication Complexity

We use communication complexity paradigm to prove our lower bound.

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One-way Two-player Communication Model:

- Alice gets a private input X and Bob gets a private input Y.
- Their goal is to compute a function $P(X,Y)$.
- Alice is allowed to send a single message M to Bob.
- Bob uses the message M plus his input to compute $f(M,Y) \approx P(X,Y)$.

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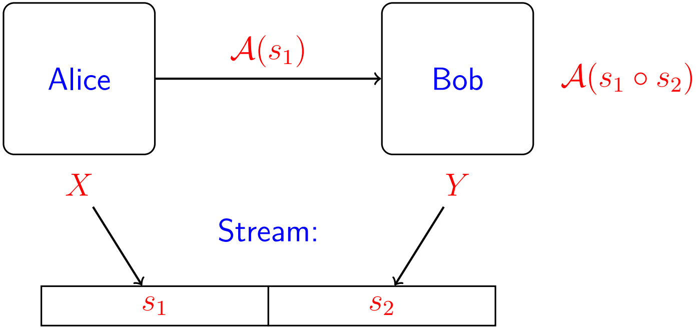
Communication Complexity $\mathsf{CC}(P)$: the minimum length of a message for any protocol that solves P with probability at least $2/3$.

Connection to Streaming Complexity

Space needed by any streaming algorithm for a problem P is at least the communication complexity of P.



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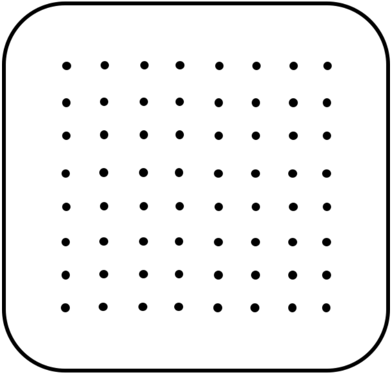
- Alice and Bob each gets a collection of sets.
- Alice sends a single message to Bob and Bob outputs an α-approximate set cover.

A Hard Input Distribution for Set Cover

Input Distribution $\mathcal{D}$:



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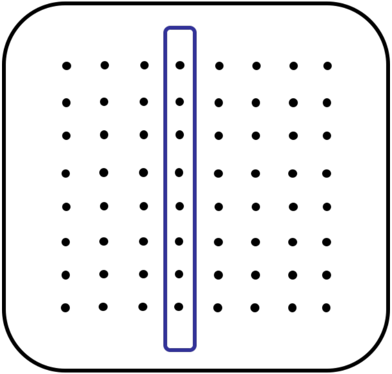
Input Distribution $\mathcal{D}$:

- Alice: near orthogonal sets of size n/α.



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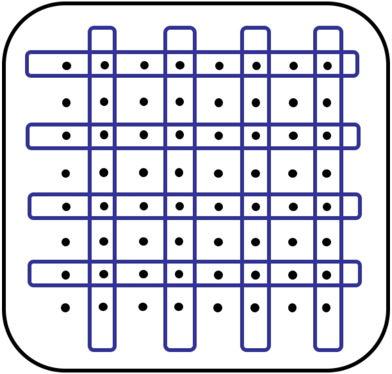

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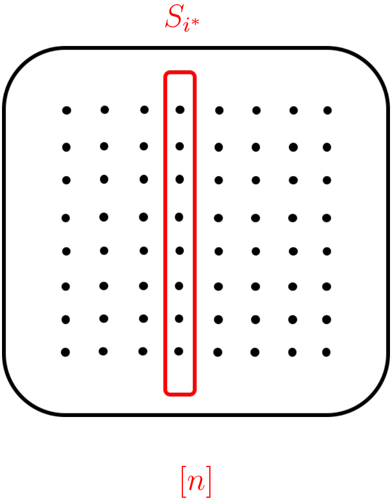
Input Distribution $\mathcal{D}$:

- Alice: near orthogonal sets of size n/α.
- Bob: a single set T of size $n - 6\alpha$:



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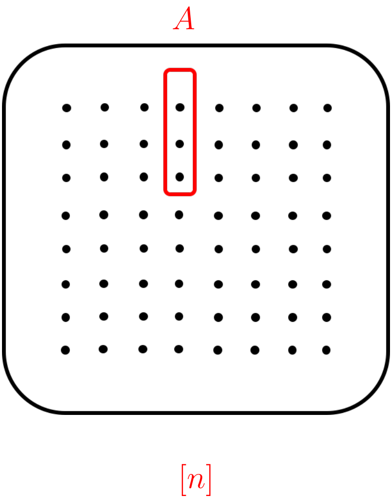

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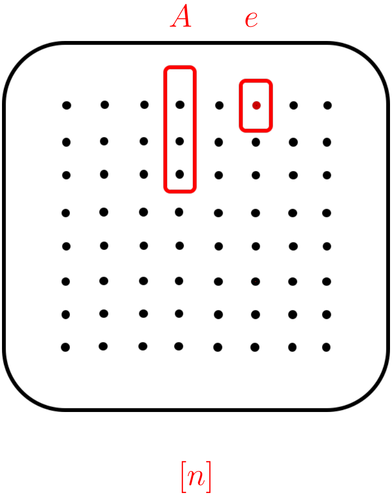

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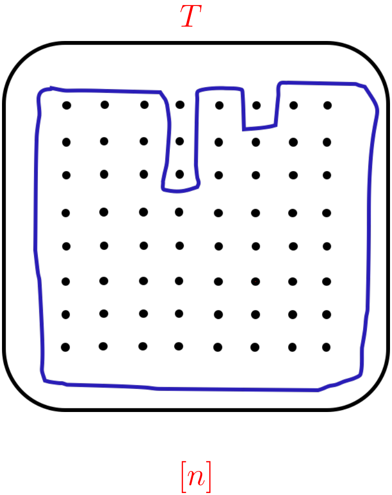

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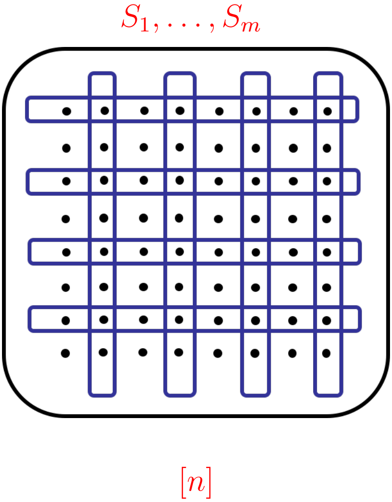

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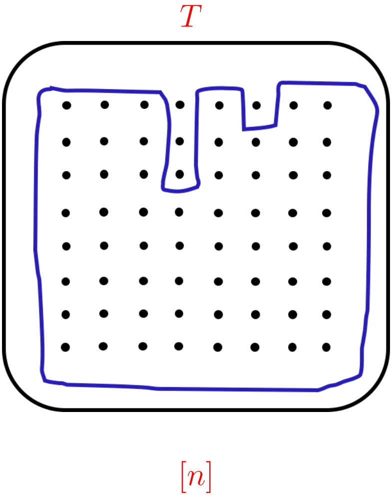

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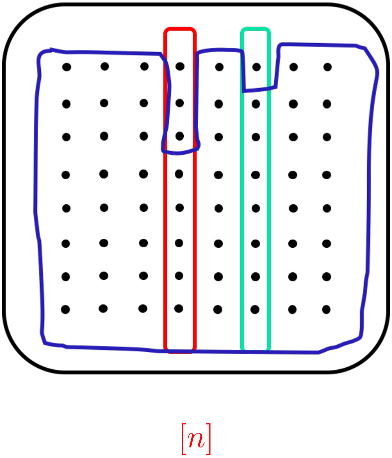
Use T,S_{i^*}, and one more set for covering the special element.



Proof Sketch

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   Identify $=$ find a small enough subset of $\overline{T}$ that contains e.

   In other words, trap the special element e.

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   In other words, trap the special element e.
4. Bob can then cover the trap-set using sets other than S_{i^*}.

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3. The near orthogonality of the sets implies that the trap-set has to be of size $< 3\alpha$.

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Why $\mathcal{D}$ is a hard distribution?

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Suppose Alice only has a single set, i.e., only S_{i^*}; then, trapping the special element requires full knowledge of Alice's set.

Trap problem: the communication problem of trapping the special element, when Alice has a single set S and Bob has a single set $A \cup \{e\}$.

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We formalize this using an information-theoretic argument and a novel reduction from the Index problem.

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Claim

When i^* is not known to Alice, trapping the special element requires m times more communication:

$$CC(\alpha\text{-Approximate Set Cover}) \approx m \cdot CC(\text{Trap})$$

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We formalize this using information complexity and a direct-sum style argument.

Summary

Hence,

$$\text{CC}(\alpha\text{-Approximate Set Cover}) \approx \Omega(mn/\alpha)$$

Communication complexity is also a lower bound on the space complexity of the streaming algorithms:

Theorem

For any $\alpha = o(\sqrt{n})$, $\Omega(mn/\alpha)$ space is necessary for α-approximating the set cover problem.

Moreover, this space-approximation tradeoff is tight.

$\widetilde{O}(mn/\alpha^2)$ Space is Sufficient for α-Estimating Set Cover

An α-Estimation Algorithm in $\widetilde{O}(mn/\alpha^2)$ Space

We show that,

Theorem

There exists a single-pass streaming that α-estimates the weighted set cover problem in $\widetilde{O}(mn/\alpha^2)$ space.

These ideas can be further generalized to estimate optimal solution value of any covering integer program.

α-Approximation in $\widetilde{O}(mn/\alpha)$ space

A simple algorithm for (weighted) set cover:

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Element Sampling

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Element Sampling:

- Sample each element with probability $1/\alpha$ and work with the sampled universe in the second phase of the algorithm.
- Store the sampled instance completely (after pruning).

  (each set has $\leq n/\alpha^2$ elements in the sampled universe and hence total space requirement is $O(mn/\alpha^2)$)

The hope is that the sampling procedure reduces the weight of the optimal set cover by a factor of at most α.

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Ideally, we also want $\mathsf{OPT}(\mathcal{I}_\alpha) \geq \mathsf{OPT}(\mathcal{I})/\alpha$ with probability $\Omega(1)$.

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But is this true?

Element Sampling

This is not true in general.

Consider the following instance $\mathcal{I}$ with n sets:

- $S_1 = \{1\}$ with weight $W \gg n$.
- $S_i = \{i\}$ for $i > 1$ with weight 1.

Clearly,

- $\mathsf{OPT}(\mathcal{I}) = (n-1) + W$
- $\Pr\left[\mathsf{OPT}(\mathcal{I}_\alpha) \geq \mathsf{OPT}(\mathcal{I})/\alpha\right] = o(1)$

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The problem is existence of elements that are too expensive to cover.

Element Sampling Lemma

- For each element $e \in [n]$, define $\mathit{Cost}(e)$ to be the minimum weight of any set that covers e.
- Define $\mathit{Cost}(\mathcal{I}) := \max_{e \in [n]} \mathit{Cost}(e)$.

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Lemma (Element Sampling Lemma)

For any instance $\mathcal{I}$, let $\mathcal{I}_\alpha$ be an instance obtained by sampling each element independently with probability $\frac{\ln(n)}{\alpha}$, then,

$$\Pr\left[\mathit{OPT}(\mathcal{I}_\alpha) + \mathit{Cost}(\mathcal{I}) \geq \frac{\mathit{OPT}(\mathcal{I})}{\alpha}\right] \geq \frac{1}{2}$$

Upper Bound Statement

Theorem

For any $\alpha = o(\sqrt{n})$, $\widetilde{\Theta}(mn/\alpha^2)$ space is sufficient for α-estimating the weighted set cover problem.

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$\widetilde{\Theta}(mn/\alpha)$ space is necessary and sufficient.

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Summary of Our Results

For the set cover problem in single-pass streams,

α-approximation:

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Our results resolve the space-complexity of set cover in single-pass streams.

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