Fully Dynamic Maximal Independent Set with Sublinear Update Time

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Joint work with:

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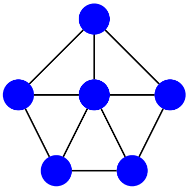
IBM Research

The Maximal Independent Set Problem

In a graph $G(V, E)$:



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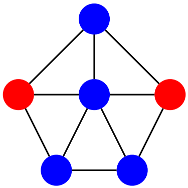

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The Maximal Independent Set Problem

Finding an MIS is a fundamental problem with numerous applications.

Closely related to a plethora of other basic problems, such as vertex cover, matching, vertex coloring, and edge coloring.

Has been studied extensively in various settings, in particular parallel and distributed algorithms.

- Initiated by seminal works of [ABI86, Lub86, Lin87].

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Maintaining an MIS in (sequential) dynamic graphs setting was left open by [CHK16].

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- Is there something better we could do?

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Let us examine a natural strategy for maintaining an MIS $\mathcal{M}$ in $O(\Delta)$ update time.

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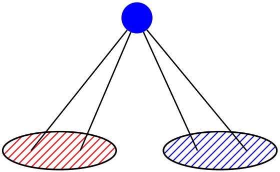
Invariant: Every vertex knows its neighbors in $\mathcal{M}$.



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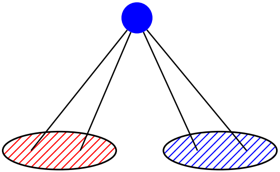
So any vertex in $O(1)$ time can decide to join or leave $\mathcal{M}$.



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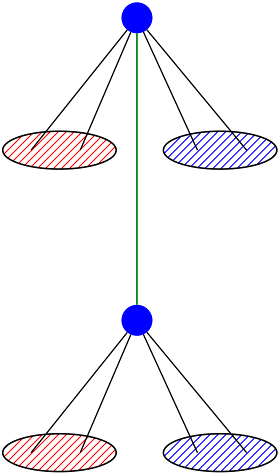
If $u, v \notin \mathcal{M}$ nothing to do.



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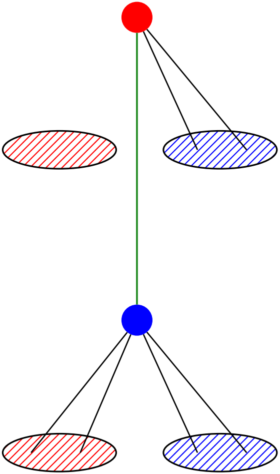

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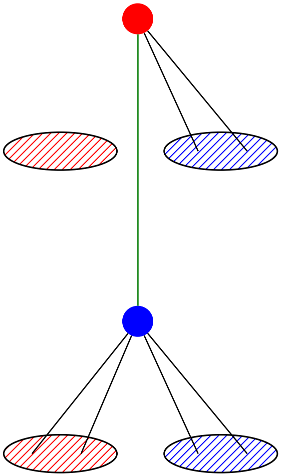
- Edge insertion: update list of $\mathcal{M}$-neighbors of v.
- Edge deletion: v checks if it can now join $\mathcal{M}$ or not. Inform all its neighbors in $O(\Delta)$ time if it joins.



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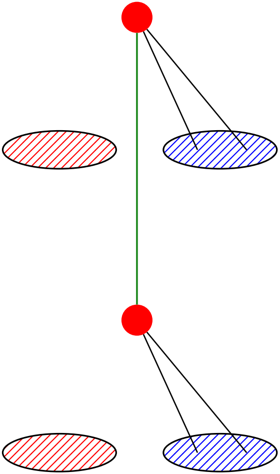

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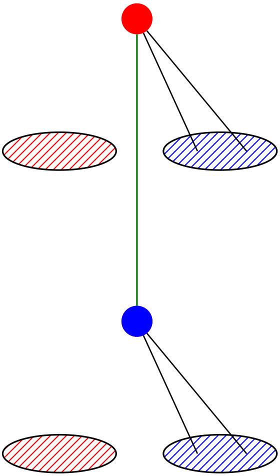

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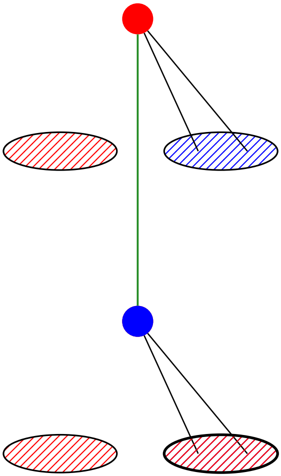
  This is problematic: $\Omega(\Delta)$ vertices potentially need to inform their $\Omega(\Delta)$ neighbors if they joined $\mathcal{M}$!



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Amortized update time is only $O(\Delta)$!

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> A maximal independent set can be maintained deterministically in $O(m^{3/4})$ amortized update time, where m denotes the dynamic number of edges.

First improvement over the $O(m)$ update time for all values of m.

Can also be implemented in distributed dynamic networks, strengthening the result of [CHK16].

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- A huge gap between the update time of best deterministic vs randomized algorithms for dynamic problems:
  - Maximal Matching: $O(\sqrt{m})$ for deterministic [NS13] vs $O(1)$ for randomized [Sol16].
  - $(\Delta + 1)$-Vertex Coloring: No non-trivial deterministic algorithm vs $O(\log \Delta)$ for randomized [BCHN18].

An $O(m^{3/4})$ Amortized Update Time Dynamic Algorithm for MIS

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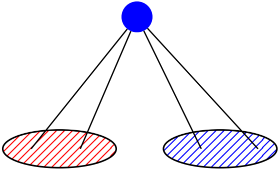

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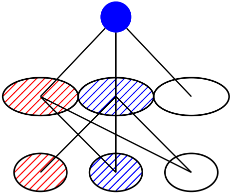

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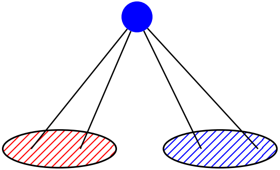

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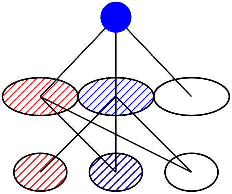

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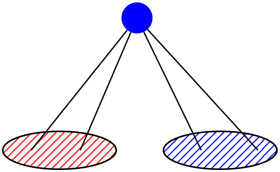

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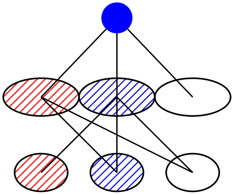

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- To compensate, we also maintain some information about 2-hop neighbors of vertices.

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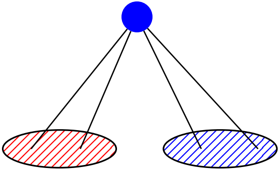

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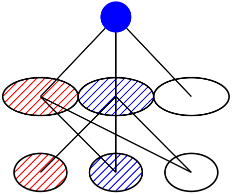

Our Algorithm: High Level Plan

Part II: Maintaining the MIS using the local and inconsistent information of vertices.

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Main challenge:

- Based on their partial information, some vertices may join $\mathcal{M}$.
- They may however have some neighbors already in $\mathcal{M}$.
- As such we may need to delete some vertices in the current $\mathcal{M}$ and process them recursively again.

Maintaining the Partial Information

Vertices are partitioned into four sets based on their degrees:

V_{High}: $deg \geq m^{3/4}$

$V_{\text{Med-High}}$: $m^{3/4} > deg \geq m^{1/2}$

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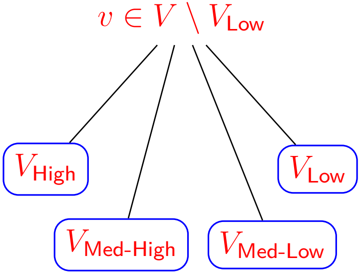

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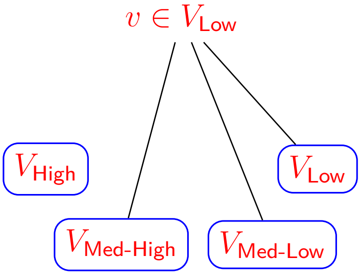

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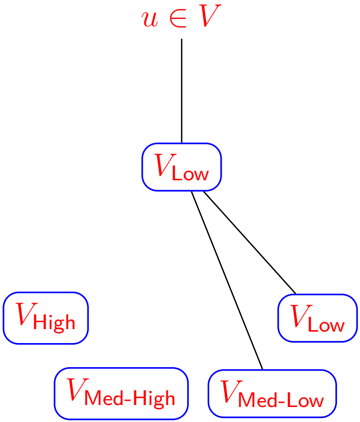
$\mathcal{M}$-2-hop-neighbor information:

- Any vertex u for any one of its neighbors v in V_{Low} knows all neighbors of v that are in $V_{\text{Low}} \cup V_{\text{Med-Low}}$ and are in $\mathcal{M}$.



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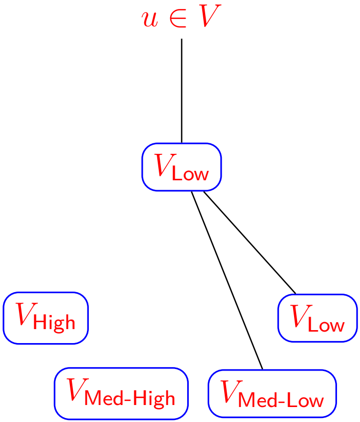

Lemma. One can maintain this information in $O(m^{3/4})$ time per each update to the graph or $\mathcal{M}$.

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This is enough to obtain the $O(m^{3/4})$ amortized update time.

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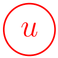
The main challenging update: adversary inserts an edge (u, v) between $u, v \in \mathcal{M}$.

- We delete u from $\mathcal{M}$.



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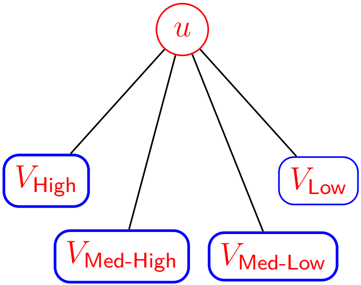

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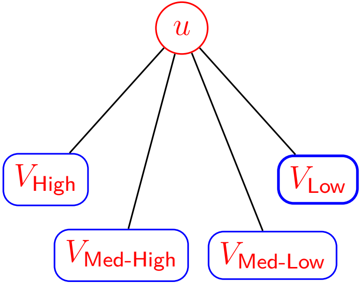
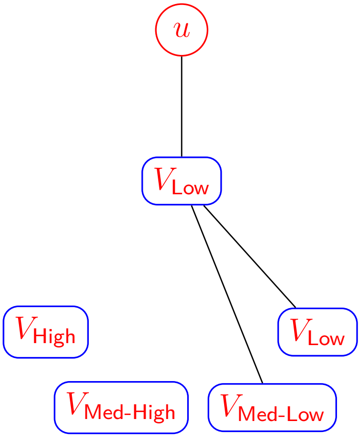
- Neighbors of u inside V_{Low}:
  - Their local information is not enough to determine whether they should join $\mathcal{M}$ or not.



Handling Low-Degree Neighbors of u

- u knows all its neighbors that do not have a neighbor in $\mathcal{M} \cap (V_{\text{Low}} \cup V_{\text{Med-Low}})$.



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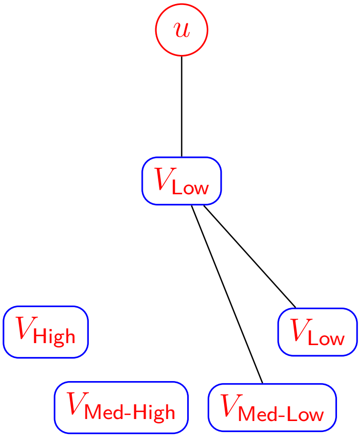

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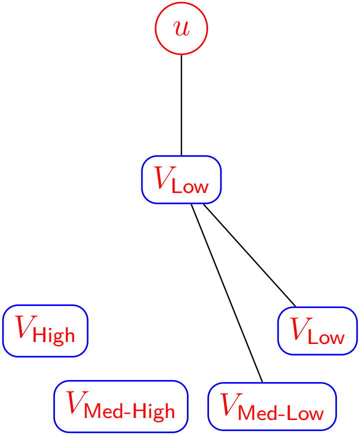
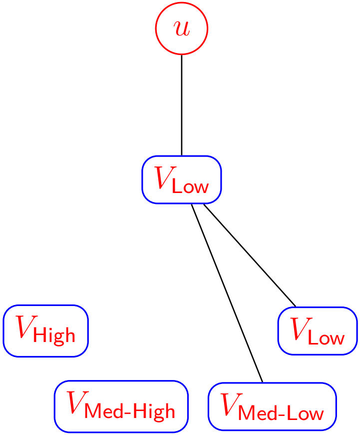
- u knows all its neighbors that do not have a neighbor in $\mathcal{M} \cap (V_{\text{Low}} \cup V_{\text{Med-Low}})$.
- Let A be the set of such neighbors.
- We process the update based on whether A is large or not.



Handling Low-Degree Neighbors of u

Let us assume A is very large i.e., has $\omega(m^{3/4})$ vertices:



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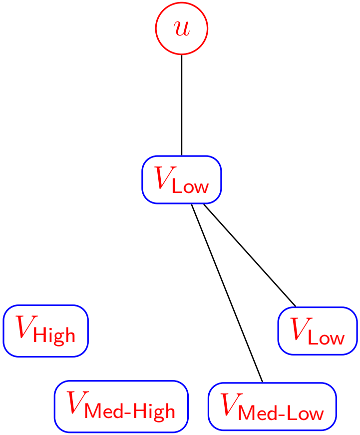

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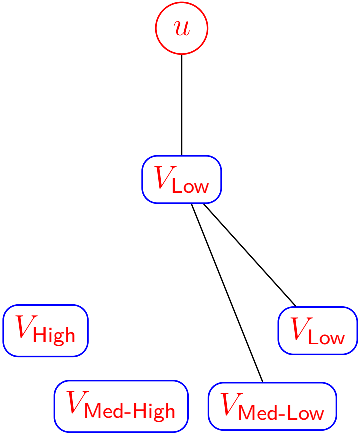

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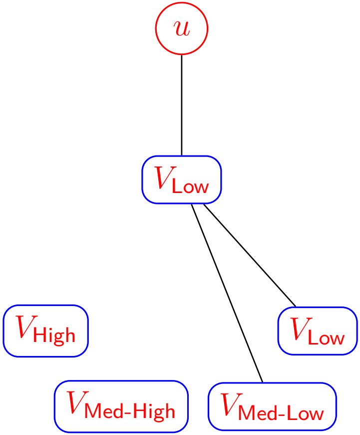

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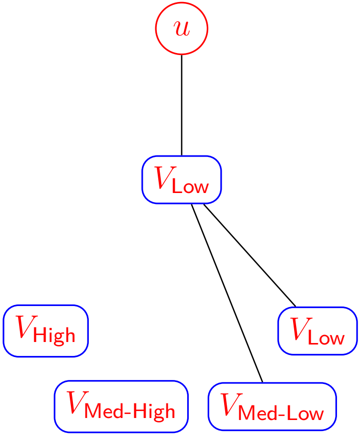

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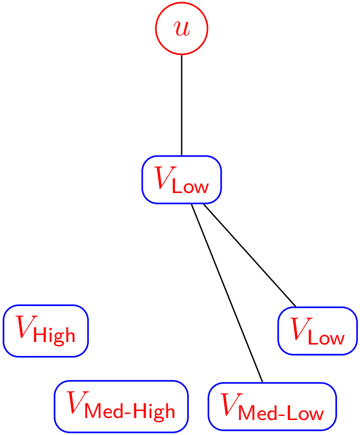
   - How many deleted? $O(m^{1/2})$.



Wrap-Up

- We spend $O(m^{3/4})$ time per update for maintaining the local information.
- We spend $O(m^{3/4})$ time for any vertex inserted to or deleted from $\mathcal{M}$.
- By main invariant, we can charge each vertex deleted from $\mathcal{M}$ with $O(m^{3/4})$ time, to be used when this vertex is inserted back later (if ever).

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MIS can be maintained deterministically with $\min\left\{O(m^{3/4}), O(\Delta)\right\}$ amortized update time in dynamic graphs.

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Thank you!

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