Combinatorial Auctions Do Need Modest Interaction

Sepehr Assadi

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Auctions and Interaction

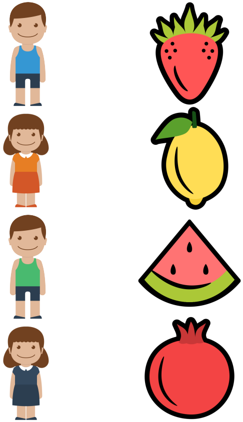
Do we need interaction between individuals in order to determine an efficient allocation?

Non-interactive

Interactive

Combinatorial Auctions

n bidders N and m items M + a central planner:



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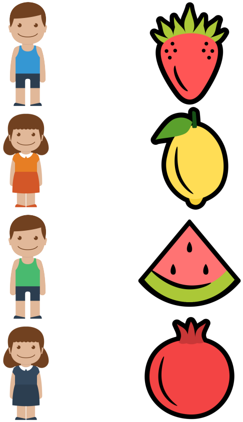

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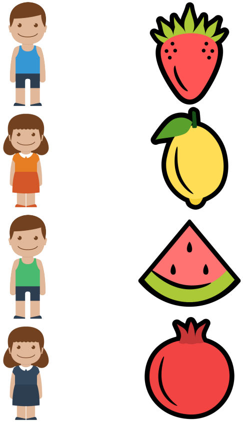
- Valuation functions are:
  - Normalized
    $v(\emptyset) = 0$
  - Monotone
    $v(A) \leq v(A \cup \{j\})$
  - Subadditive
    $v(A \cup B) \leq v(A) + v(B)$



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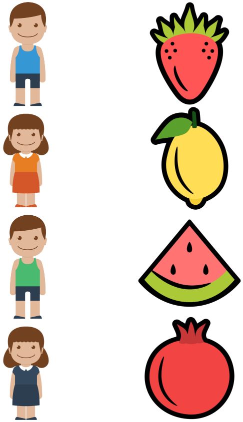
  - Subadditive $v(A \cup B) \leq v(A) + v(B)$
- Find an allocation $(S_1, \ldots, S_n)$ that maximizes social welfare $\sum_{i \in N} v_i(S_i)$.



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We are interested in protocols with $\mathrm{poly}(m, n)$ communication cost.

Non-Interactive Protocols

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1. Bidders communicate their entire inputs.
   Pros. Exact answer.
   Cons. Exponential communication.

2. Bidders communicate a poly-size representation of their inputs.
   Pros. Polynomial communication.
   Cons. Approximation ratio is $\Omega(\sqrt{m})$ [Badanidiyuru et al., 2012].

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Thm [Dobzinski et al., 2014]. Any non-interactive $\mathrm{poly}(m,n)$-communication protocol has an approximation ratio $\Omega(m^{1/4})$.

Interactive Protocols

Many interactive constant-factor approximation protocols are known for this problem [Dobzinski et al., 2005] [Dobzinski and Schapira, 2006] [Feige, 2009] [Feige and Vondrák, 2006] [Lehmann et al., 2006] [Vondrák, 2008] . . .

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In particular,

Thm [Feige, 2009]. There exists an interactive 2-approximation protocol with $\mathrm{poly}(m, n)$ communication.

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Do we need interaction between individuals in order to determine an efficient allocation? Yes!

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Auctions and Interaction

How much interaction do we need between individuals in order to determine an efficient allocation?

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Auctions and Interaction

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Interactivity should be thought of as a wide spectrum!

Combinatorial Auctions with Limited Interaction

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Indeed, even a round-approximation tradeoff is known!

Thm [Dobzinski et al., 2014]. For any $r \geq 1$, there exists an r-round $\widetilde{O}(r \cdot m^{1/r+1})$-approximation protocol with polynomial communication.

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> *In (subadditive) combinatorial auctions, logarithmic number of rounds suffices to obtain an (almost) efficient allocation with polynomial communication.*

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Similar question has been studied previously in the context of unit-demand auctions (orthogonal to our setting):

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The case of subadditive combinatorial auctions was posed as an open problem by [Dobzinski et al., 2014] and [Alon et al., 2015].

Our Results

In a nutshell:

In (subadditive) combinatorial auctions, logarithmic number of rounds is also necessary to obtain an (almost) efficient allocation with polynomial communication.

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Theorem

Any $\mathrm{polylog}(m)$*-approximation protocol for subadditive (even XOS) combinatorial auctions that uses polynomial communication requires* $\Omega(\frac{\log m}{\log\log m})$ *rounds of interaction.*

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In fact, we prove an almost tight round-approximation tradeoff.

Theorem

For any integer $r \geq 1$*, any* r*-round protocol for subadditive combinatorial auctions that uses polynomial communication can only achieve an* $\Omega(\frac{1}{r} \cdot m^{1/2r+1})$ *approximation.*

A Lower Bound for r-Round Protocols

We design a hard input distribution $\mathcal{D}_r(n_r, m_r)$ with

- $n_r = k^{2r}$ bidders
- $m_r \approx k^{2r+1}$ items
- Yes case: social welfare $= k^{2r+1}$.
- No case: social welfare $< k^{2r+\varepsilon}$ for every constant $\varepsilon > 0$.

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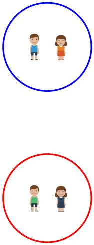
- Distinguishing between Yes and No cases requires $\exp(k)$ communication.
- A round-elimination argument: Distinguishing Yes and No cases in distribution $\mathcal{D}_r(n_r, m_r)$ in r rounds is hard as in distribution $\mathcal{D}_{r-1}(n_{r-1}, m_{r-1})$ in $r-1$ rounds.

A Hard Distribution for r-Round Protocols

- The bidders are arbitrary partitioned into k^2 blocks each of size n_{r-1}.



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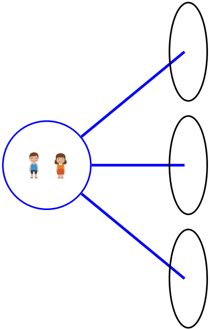
- The bidders are arbitrary partitioned into k^2 blocks each of size n_{r-1}.
- The bidders in each block are playing in $\exp(k)$ many instances of $(r-1)$-round problem, each over m_{r-1} items.



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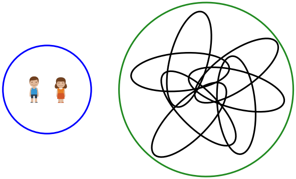

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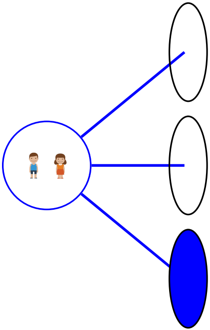

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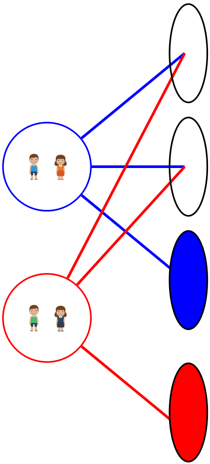

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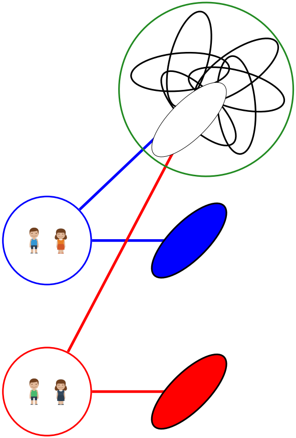

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Analysis Sketch for r-Round Protocols

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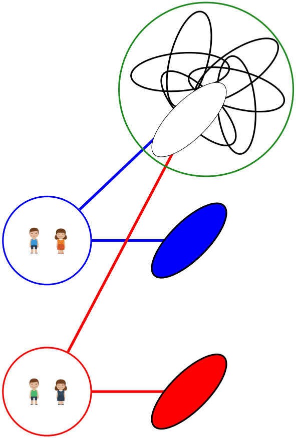

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1. The bidders need to solve the $(r-1)$-round special instance in their group.
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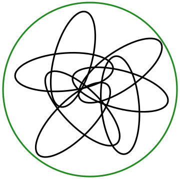

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Analysis Sketch for r-Round Protocols

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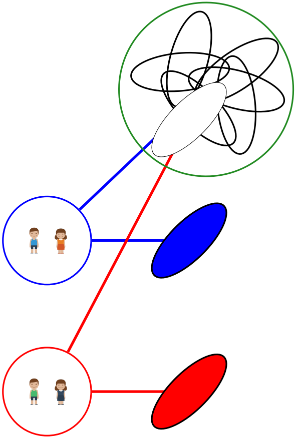
2. The first message M of a $\mathrm{poly}(m,n)$-cost protocol π does not reveal any useful information about the special instance.
3. If π can solve $\mathcal{D}_r$ in r rounds, then $\pi \mid M$ should be able to solve $\mathcal{D}_{r-1}$ in $r-1$ rounds.



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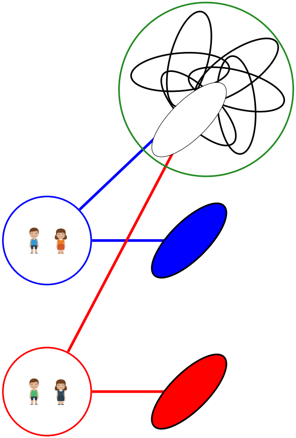
4. We can obtain a $\mathrm{poly}(m,n)$-cost protocol π' for solving $\mathcal{D}_{r-1}$ in $r-1$ rounds by simulating $\pi \mid M$.



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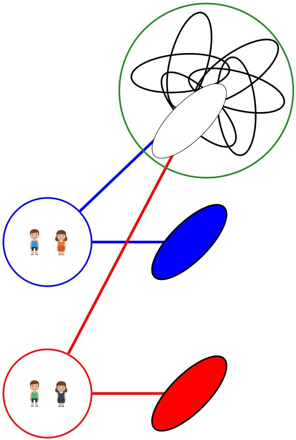
4. We can obtain a $\mathrm{poly}(m,n)$-cost protocol π' for solving $\mathcal{D}_{r-1}$ in $r-1$ rounds by simulating $\pi \mid M$. Contradiction!



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Concluding Remarks

[Dobzinski et al., 2014]:

> *A modest amount of interaction between individuals is sufficient for obtaining an efficient allocation.*

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Open problems.

- More restricted classes of valuation functions, e.g., submodular?
- Tightening the gap for unit-demand bidders?
  - $\Omega(\log\log m)$ lower bound in [Alon et al., 2015] vs $O(\log m)$ upper bound in [Dobzinski et al., 2014].

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