Only for the personal use of students registered in CS 671, Fall 2020 at Rutgers University. Redistribution out of this class without the Instructor's permission is not permitted.

CS 671: Graph Streaming Algorithms and Lower Bounds **Rutgers: Fall 2020**

Problem set 3

Due: 11:59PM, September 29, 2020

Please solve *exactly one* of the problems below.

Problem 1. For a graph $G = (V, E)$, the *degeneracy* of G, denoted by $\kappa(G)$, is defined as follows: Let π be any permutation of vertices and let $k(\pi)$ denote the maximum degree of any vertex v to vertices that appear *before* v in π, i.e.,

$$k(\pi) := \max_{v \in V} |\{(u, v) \in E \mid \pi(u) < \pi(v)\}|.$$

Then, the degeneracy of G is defined as the minimum possible value of $k(\pi)$, i.e.,

$$\kappa(G) := \min_{\pi} k(\pi).$$

(*i*) Prove that any graph $G = (V, E)$ can be properly colored with $\kappa(G) + 1$ colors (considering $\kappa(G) \leq \Delta$ always, this gives a strengthening of the $(\Delta + 1)$ coloring).

(*ii*) Prove a Palette Sparsification Theorem for $2 \cdot \kappa(G)$ coloring of G, i.e., prove the following statement:

> Suppose for every vertex $v \in V$, we sample $O(\log n)$ colors $L(v)$ from $\{1, \ldots, 2 \cdot \kappa(G)\}$ independently and uniformly at random; then, with high probability, G can be list-colored from lists $\{L(v) \mid v \in V\}$.

Problem 2. Consider the following coloring *verification* problem: You are given a single pass over the edges of the graph $G = (V, E)$ with maximum degree Δ; *At the end of this pass*, you will be given a $(\Delta+1)$ coloring $c : V \to \{1, \ldots, \Delta + 1\}$ of vertices, and should output whether this is a proper coloring of G or not, i.e., is it the case that $c(v) \neq c(u)$ for all edges $(u, v) \in E$. Prove that this problem requires $\Omega(n^2)$ space.