

Problem set 11

Due: 11:59PM, November 24, 2020

Problem 1. In Lecture 11, we saw a two-pass semi-streaming algorithm for finding an exact (global) minimum cut of *unweighted* undirected graphs. In this question, we extend this result to weighted graphs with “small weights”.

Let $G = (V, E, w)$ be a weighted undirected graph with an integer weight function $w : E \rightarrow \{1, \dots, W\}$. The goal is to find a cut $(S, V \setminus S)$ of G with minimum weight of crossing edges, i.e.,

$$\arg \min_S \sum_{e=(u,v):u \in S, v \in V \setminus S} w_e.$$

Design a two-pass streaming algorithm for this problem that uses $O(n \cdot \text{poly}(W, \log(n)))$ space¹ (this will give a semi-streaming algorithm for weighted graphs with weights bounded by $\text{polylog}(n)$).

Hint: Simply use the same sparsification approach in Lecture 11 for this problem also (recall that we already discussed how to obtain a cut sparsifier for weighted graphs—you can simply assume it in this question). Prove a generalization of the key structural lemma we used in the lecture that bounds the number of edges participating in “small” cuts; to do this, it helps to first figure out which part of the proof crucially used the fact that the graph was a simple graph with no weights and parallel edges².

¹A back of envelope calculation suggests you might be able to get an $O(nW \cdot \text{polylog}(n))$ or perhaps even $O(n\sqrt{W} \cdot \text{polylog}(n))$ space, however for this question, any polynomial dependence on W is fine.

²*Double hint:* in a simple graph, bounding the number of vertices in a component also bounds the number of edges inside the component; this is no longer true for weighted graphs (but a weaker bound based on W still holds that you can use).